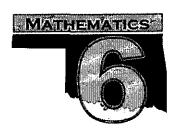
Chapter 7 Introduction to Algebra



7.1 Algebra Vocabulary

Algebra is the branch of mathematics that uses a combination of letters, numbers, and operations (addition, subtraction, multiplication, and division) to show how two or more things are related to each other. Solving algebra problems can be made simple if you learn the language and rules. Algebra can also be used to solve problems that arise in real life. The table below shows the basic vocabulary for solving algebra problems.

Vocabulary Word	Example	Definition
expression	4x + 3	a mathematical combination of numbers, variables, and operations
variable	4x (x is the variable)	a letter that can be replaced by a number
coefficient	4x (4 is the coefficient)	a number multiplied by a variable or variables
term	$5x^2 + x - 2$ (5 x^2 , x, and 2 are terms)	numbers or variables separated by $+$ or $-$ signs
constant	5x + 2y + 4 (4 is a constant)	a term that does not have a variable
sentence	$2x = 7 \text{ or } 5 \le x$	two algebraic expressions connected by =, \neq , <, >, \leq , \geq , or \approx
equation/equality	4x = 8	a sentence with an equal sign
inequality	$7x < 30 \text{ or } x \neq 6$	a sentence with one of the following signs: \neq , <, >, \leq , \geq , or \approx
solution	if $3x = 9$, then $x = 3$	numbers that will make a sentence true
base	6^3 (6 is the base)	the number used as a factor
exponent	6^3 (3 is the exponent)	the number of times the base is multiplied by itself

Use the vocabula	ary words below to	complete the pa	iragraph. Some	words will be	used more
	nay have to add or				

1. can be used to solve real world mathematical problems and	situations. Solving 1.
problems can be made simple by learning to use a few new words.	1. uses symbols, integers.
and letters to represent mathematical ideas. This combination of syn	abols, integers, and letters are
called 2 2. are like instructions for solving a puzzle	
part of the 2 , separated by $+$ and $-$ signs, are actually detailed	
final answer. For example, "52" is one of the <u>3.</u> in the <u>2.</u>	
to the second power" or "5 squared". This tells me that I must multi	nly 5 the A by itself
because 2 is the After I complete that step, then I only n	end to add 7 to find my final
answer of 32. When I write $5^2 + 7 = 32$, it is no longer a(n) 2	tit is now of
This 6 is called $a(n) = 7$ because of the size Some	, it is now a(n)
This <u>6.</u> is called a(n) <u>7.</u> because of the = sign. Some	
because they describe two 2. and one is either greater than ()	
This type of <u>6.</u> is known as a(n) <u>8.</u> Sometimes letters a	ire used to describe unknown
amounts. These letters are called 9. If the 9. has a number of the 16 in the	mber in front of it, as in $16x$,
then 16 is the $\underline{10}$ of the $\underline{9}$, x .	
1.	al a alama
1.	algebra
2.	equation(s)
2.	equation(s)
3.	inequality
	. ,
	,
4.	base
	••
5.	equality
6.	avenagai an(a)
0.	expression(s)
7.	coefficient
•	
8.	exponent
	•
9.	variable(s)
10	tomm(a)
10.	term(s)

7.2 Understanding Algebra Word Problems

Algebra word problems are used to describe real life situations and solve real life problems. The key to correctly solving a word problem is to correctly express the verbal ideas in algebraic form. This is where the word "expression" gets its meaning in algebra. A simple list of what to do will help you solve algebra word problems. Vocabulary will help you determine which operation should be completed to correctly answer the problem.

Do

- 1. Read the problem CAREFULLY.
- 2. Decide what is known. These terms are expressed as constants (integers).
- 3. Decide what is unknown. These terms are expressed as variables. (Whether or not the variable has a coefficient is based on each problem.)
- 4. Decide what the question is asking. Determine the operation needed to be done.

The list of keywords below will help you identify the operation that needs to be done.

Operation	Keywords
Addition	increased by, is added, totals, combined, more than, plus, more, sum, and
Subtraction	difference, subtracted from, subtracted by, decreased by, minus, less, lower than
Multiplication	times, each, of, double, twice, half, multiply, product, triple
Exponents	squared (to the second power), cubed (to the third power), to the fourth power, etc.
Division	divided by, quotient, divided into, divided among, ratio of/to

Use what you have learned to match each verbal expression to the letter of its matching algebraic expression. Some letters will be used more than once.

1. a number less four	A. $x - 4$
2. a number multiplied by four	
3. a number to the fourth power	B. $x + 4$
4. a number divided by four	C. $\frac{x}{4}$
5. four more than a number	4
6. four times a number	D. x^4
7. x increased by four	
8. four less than a number	E. 4x

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Write an algebraic expression for each phrase below.

- 9. five guests more than planned
- 10. the class with eight students missing
- 11. a number decreased by thirty-one
- 12. the difference of a number and eighteen
- 13. Eight dollars per hour
- 14. the product of eight and the third power of a number
- 15. nine dollars minus purchases
- 16. eighty percent of a number
- 17. the total number of cupcakes divided among four trays
- 18. half the number of cookies plus seven extra
- 19. bacteria culture, b, doubled
- 20. triple John's age, y
- 21. n feet lower than 10
- 22. 3 more than p
- 23. the product of 4 and m
- 24. a number, y, decreased by 20
- 25. 5 times as much as x
- 26. a number, n, plus 4
- 27. quantity, t, less 6
- 28. 18 divided by a number, x

7.3 Setting Up Algebraic Expressions and Equations from Verbal Statements

So far you have named terms and created expressions. The next step is to compare two expressions. The two expressions can be equal to (=), greater than (>), or less than (<) each other. Two expressions that are "equal to" each other make an **equation**. Two expressions that are either greater than or less than each other are called **inequalities**. Listed below are the keywords to help you identify how to compare the expressions in both equations and inequalities.

Symbol Keywords

Equations = equals, is, was, were, are, equivalent to, gives, yields, totals Inequalities <, > less than, greater then

Remember to read each statement completely and carefully.

Express each written statement algebraically. Don't solve the problems.

- 1. four less than a number is fifteen
- 2. six divided by a number is twelve
- 3. eleven is less than four times a number
- 4. a number squared is less than the same number cubed
- 5. a number and another number total sixteen
- 6. the difference of a number and eight is equivalent to six
- 7. a number increased by nine is greater than a different number plus three
- 8. nine and a number equal seventeen
- 9. ninety minus a number is ten
- 10. four combined with a number squared yields twenty-nine

7.4 Changing Algebra Word Problems to Algebraic Equations

Now we will practice what we have learned to write algebraic equations for actual word problems. Simply focus on correctly writing the equation. You will learn strategies to solve these equations in later chapters.

Example 1: Triple Maddie's age is less than fifty.

Known: fifty (50)

Unknown: Maddie's age; we'll call this m.

What must be done? Triple m: 3m.

Keywords: is less than (<)

Compare unknowns to knowns: 3m < 50

Example 2: Find the number that is the sum of four squared and six squared.

Let x = the number

Keyword: sum (+); is (=)

Known: 4^2 , 6^2

Compare: $x = 4^2 + 6^2$

Example 3: Molly treated 5 girlfriends and herself to smoothies for \$4.95 each. The cashier charged her an additional \$1.78 in sales tax. She left the store with \$6.28. How much money did Molly start with?

Known: Molly + 5 friends (\$4.95) for smoothies; \$1.78 sales tax, \$6.28 left over

Unknown: Molly's starting amount; let x = Molly's starting amount Compare: 6(\$4.95) + 1.78 + 6.28 = x or x - 6(\$4.95) + 1.78 = 6.28

Example 4: There are 3 kids in Sally's family. Sally is three years younger than Joe. Rob is 2 years older than Joe. The kids' ages combined total 27. How old is each kid? We know how each of them relates to Joe's age, so we can write each term as it relates to Joe's age.

Joe Sally is three years younger than Joe. Rob is two years older than Joe.

 $Joe = j \quad Sally = j - 3 \qquad \qquad Rob = j + 2$

Now we can find their ages because we know that Joe + Sally + Rob = 27. So, to solve this problem we can write i + j - 3 + j + 2 = 27.

Example 5: The mall has all jackets on sale for 30% off. How much would be saved on a \$120 jacket?

Known: price of jacket, \$120

Unknown: savings, s

What must be done? Find 30% of 120. (Multiply 120 by 30%.)

Compare: 30%(120) = s or 0.30(120) = s

Set up the following words problems as equations.

- 1. The total weight of 3 football players is 700 pounds. Mark weighs m pounds. Jon weighs 50 pounds more than Mark. Ray weighs 50 pounds less than Jon.
- 2. Fluffy, Spot, and Shampy have a combined age in dog years of 91. Spot is 14 years younger than Fluffy. Shampy is 6 years older than Fluffy. What is Fluffy's age, f, in dog years?
- 3. Julia bought a coat for 25% off \$140. What was her savings, s?
- 4. Write an equation to show the sales price, p, of Julia's coat.
- 5. If a 6% sales tax was added to Julia's coat purchase, how would the sales tax, t, be written as an equation?
- 6. Tickets for the homecoming dance were sold for three days. The first day, t number of tickets were sold. The second day, the number of tickets sold increased by 20 over the first day. The third day, ticket sales doubled the first day. The total number of tickets sold was 420. Write an equation to show how the tickets sold each day add to 420.
- 7. If 500 tickets were printed for the dance, write an equation to show how many are left, x.
- 8. Beverly makes \$8 per hour plus a commission, c, for all sales during her shift. Write an equation to show her total pay, p, for 6 hours plus any commissions.

7.5 Arithmetic Properties

The Associative, Commutative, and Distributive properties are listed below by example as a quick refresher.

Property

Example

$$(a+b)+c=a+(b+c)$$

$$(a \times b) \times c = a \times (b \times c)$$

$$a+b=b+a$$

$$a \times b = b \times a$$

$$a \times (b+c) = (a \times b) + (a \times c)$$

Write the number of the property listed above that describes each of the following statements.

$$1.354 + 453 = 453 + 354$$

$$2.5 + (7 + 12) = (5 + 7) + 12$$

$$3.3+9=9+3$$

4.
$$(4 \times 5) \times 2 = 4 \times (5 \times 2)$$

5.
$$r \times z = z \times r$$

$$6. m(n+p) = mn + mp$$

7.
$$(x)(y \cdot z) = (x \cdot y)(z)$$

$$8. 25 (3 + 2) = (25) (3) + (25) (2)$$

$$9.6 + (7 + 8) = (6 + 7) + 8$$

10.
$$x(a-q) = (x)(a) - (x)(q)$$

Understanding Exponents 7.6

Sometimes it is necessary to multiply a number by itself one or more times. For example, a math problem may need to multiply 3×3 or $5 \times 5 \times 5 \times 5$. Instead of writing 3×3 , you can write 3^2 , or instead of writing $5 \times 5 \times 5 \times 5$, 5^4 means the same thing. The first number is the base. The small, raised number is called the exponent or power. The exponent tells how many times the base should be multiplied by itself.

744 exponent (or power) This means multiply by 7 four times: $7 \times 7 \times 7 \times 7$

$$8^6 = 8 \times 8 \times 8 \times 8 \times 8 \times 8$$

You also need to know two special properties of exponents:

- 1. Any base number raised to the exponent of 1 equals the base number.
- 2. Any base number raised to the exponent of 0 equals 1.

$$5^1 = 5$$

$$20^1 = 20$$
 $17^1 = 17$ $5^0 = 1$ $20^0 = 1$ $17^0 = 1$

$$17^1 = 17$$

$$5^0 = 1$$

$$20^0 = 1$$

$$17^0 = 1$$

Rewrite the following problems using exponents.

Example 9: $4 \times 4 \times 4 \times 4 = 4^4$

$$1.6 \times 6 \times 6 =$$

5.
$$16 \times 16 \times 16 \times 16 =$$

$$2.1 \times 1 \times 1 \times 1 \times 1 =$$

$$6.30 \times 30 \times 30 \times 30 \times 30 = 10.7 \times 7 =$$

$$3.14 \times 14 =$$

$$7.8 \times 8 =$$

$$11.\,15\times15\times15\times15\times15=$$

$$4.9 \times 9 \times 9 =$$

8.
$$11 \times 11 \times 11 =$$

$$12.13 \times 13 \times 13 =$$

Find what product each number with an exponent represents.

Example 10: $5^3 = 5 \times 5 \times 5 = 125$

13.
$$(7)^2 =$$

$$16.3^0 =$$

19.
$$2^5 =$$

22.
$$6^3 =$$

$$14.4^4 =$$

$$17. 10^3 =$$

$$20.4^2 =$$

$$23.\ 27^{1} =$$

$$15.9^1 =$$

$$18.8^2 =$$

$$21.14^0 =$$

$$24.9^2 =$$

Express each of the following numbers as a base with an exponent.

Example 11: $36 = 6 \times 6 = 6^2$

$$25.64 =$$

$$28.343 =$$

$$34.\ 1000 =$$

$$36.144 =$$

7.7 **Order of Operations**

In long math problems with +, -, \times , \div , (), and exponents in them, you have to know what to do first. Without following the same rules, you could get different answers. If you will memorize the silly sentence, Please Excuse My Dear Aunt Sally, you can memorize the order you must follow.

"P" stands for parentheses. You must do the operation inside the parentheses first.

Examples:
$$7(2+3) = 7(5) = 35$$

 $2(24-4) = 2(20) = 40$

"E" stands for exponents. You must evaluate the exponents next.

Example:
$$7^2 = 7 \times 7 = 49$$

"M" stands for multiply. "D" stands for divide. Start on the left of the equation or expression and perform all multiplications and divisions in the order in which they appear.

Aunt Sally

"A" stands for add. "S" stands for subtract. Start on the left and perform all additions and subtractions in the order they appear.

Example	12:	24	÷ 3 ((5 - 3)	$) + 2^2$	-5

Please

Do the operation inside the **parentheses**. 5-3=2 $24 \div 3 \times 2 + 2^2 - 5$

Excuse

Evaluate **exponents**. $2^2 = 4$ so now we have

 $24 \div 3 \times 2 + 4 - 5$

My Dear

Multiply and divide next in order from left to right. $24 \div 3 = 8$ then $8 \times 2 = 16$

Aunt Sally

Last, add and subtract in order from left to right.

$$16 + 4 - 5 = 15$$

Simplify the following problems.

$$1.6 + 5 \times 4 - 3 =$$

$$7.(3+7-5)\times 2=$$

$$13.3^2 + (7+1) - 5 =$$

2.
$$7 + 10^2 \times 3 - 54 =$$

$$8.\ 4\times3\div2\times8=$$

$$14.\,9^2 + 2 - 8 \times 4 =$$

3.
$$54 \div 6 - 2 \times 2 =$$

$$9.3^3 \div 9(1+2) =$$

$$15.2^2 + (4-1) \times 4 =$$

4.
$$8(4-3)+3^2=$$

10.
$$7 + 4(2 \times 6) - 1 =$$

$$16.4 - (3-6) + 2 =$$

$$5.5(2+3)-4^2=$$

$$11.60^0 - 1 + 10 \div 2 =$$

$$17.4^2(3+4) - 70 =$$

6.
$$5(12-6) \times 2 =$$

12.
$$(30 \div 3) \times 2 - 7 =$$

18.
$$3 \times 5 + 2 \times 4 =$$

When a problem has a fraction bar, simplify the top of the fraction (numerator) and the bottom of the fraction (denominator) separately using the rules for order of operations. You treat the top and bottom as if they were separate problems. Then simplify the fraction to lowest terms.

Example 13:
$$\frac{3(7-5)-2}{4^2-2(3+1)}$$

Please Eliminate parentheses.
$$(7-5) = 2$$
 and $(3+1) = 4$ $\frac{3(2)-2}{4^2-2(4)}$

Excuse Eliminate exponents.
$$4^2 = 16$$

$$\frac{3(2) - 2}{16 - 2(4)}$$

My Dear Multiply and divide in the numerator and denominator separately.
$$2(4) = 8$$
 and $3(2) = 6$ $6-2$ $16-8$

Aunt Sally Add and subtract in the numerator and denominator separately.
$$6-2=4$$
 and $16-8=8$

Now simplify the fraction to lowest terms. $\frac{4}{8} = \frac{1}{2}$

Simplify the following problems.

1.
$$\frac{4(2-1)}{(7\times 9)-3^3} =$$
 6. $\frac{(6-2)+4^2}{3(2+6)} =$ 11. $\frac{6^2-2\times 11}{7+2^3} =$

2.
$$\frac{3^2+11}{2(5+5)} =$$
 7. $\frac{(4\times10)+2}{8^2} =$ 12. $\frac{7^2-5^2+2}{9+(2^3-1)} =$

3.
$$\frac{9^2 - (7 \times 3)}{5(10 + 7) - 9^2} =$$
 8. $\frac{6 - 2^2 + 7}{5^3 - (9^2 + 17)} =$ 13. $\frac{(4 + 2) \times 7}{3^2 + (4^2 + 1)} =$

4.
$$\frac{6+3(2+1)}{2(12+3)} =$$
 9. $\frac{8\times(2+1)}{6^2-(4\times8)} =$ 14. $\frac{7^2+3}{2^3+6\times8} =$

5.
$$\frac{20 - (14 - 2)}{7^2 - 3 \times 11} = 10. \frac{(10 - 2) \times 8}{3^3 + 2^3 - 1} = 15. \frac{7 - 1^5 + 4}{4^2 - (1 + 3)} =$$

7.8 More Order of Operations

The sentence, "Please Excuse My Dear Aunt Sally", can help you remember the **order of operations** that you must follow.

Example 14: Give the <u>correct</u> order of operations to evaluate the expression $14 \div (4+3) - 2$.

- Step 1: Using the rules for order of operations, we know that you must do any operations inside the parentheses first. In the expression $14 \div (4+3) 2$, the first operation to do is addition.
- Step 2: Since there are no exponents, the next operations to do are multiplication and division. Remember to do these from left to right. In the expression, we have division. So the second operation to do is division.
- Step 3: Next, addition and subtraction must be done from left to right. In the expression, the third operation to do is subtraction.
- Answer: The correct order of operation to evaluate the expression $14 \div (4+3) 2$ are $+, \div,$ and -.

Give the correct order of operations to evaluate the following expressions.

1.
$$10 + 17 \times 4 - 16$$

$$2.12(6+4)-2$$

3.
$$15(10-3)-7 \div 2$$

$$4.22 \div 11 - 1 \times 13$$

$$5.19 \times 10 - (19 - 6)$$

6.
$$9 + 9 \div 3 \times 4$$

7.
$$2 \times 3 \div 2(2+2)$$

$$8.13 + 4(6-5) \div 8$$

Chapter 7 Review

Fill each blank with a vocabulary word.

1.	In	5b	+	9.	5	is	the		

2. In
$$6x - 17$$
, 17 is the _____.

3. In
$$4.6p - 1.4$$
, p is the _____.

4.
$$\ln x^2$$
, 2 is the _____.

5. In
$$x^5$$
, x^3 , and x^2 , x is the _____.

Write an equation for each word problem.

- 6. Jane is baking muffins. The recipe calls for 6 eggs. She has already put 2 eggs in the mixture. How many more must she add?
- 7. Clay and his 3 friends divide their dinner bill evenly. If each boy paid \$13, what was their total bill?
- 8. How many CD's can you buy with a \$60 gift card if each CD costs \$15?
- 9. Twenty-eight percent of the employees at the tire factory are women. If there are 500 employees, how many are women, w?
- 10. A furniture store advertises a 40% off liquidation sale on all items. What would the sale price, p, be on a \$2530 dining room set?
- 11. Carol got \$40 for her birthday. Now she has \$29. How much did she spend?
- 12. You and your best friend equally split the cost of a birthday cake, paying \$14 each. How much was the cake?
- 13. You want an mp3 player that costs \$129. If sales tax is 7%, what will be your total, t?
- 14. Sal is making cheesecake. He has added 1 cup of sugar already. If he needs a total of 6 cups, how many more must he add, s?
- 15. Brittany sold a painting at auction. The second bidder outbid the first bidder by \$65. Write an expression to show the second bidder's offer.

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Choose which example fits each property.

Property

- 16. Associative Property of Addition
- 17. Distributive Property
- 18. Commutative Property of Multiplication

Example

A)
$$a \times b = b \times a$$

B)
$$(a+b)+c=a+(b+c)$$

C)
$$a \times (b+c) = (a \times b) + (a \times c)$$

Solve the following problems.

19.
$$9(3+7)-12=$$

$$20.4^2 + (9 \div 3) =$$

21.
$$(18 \div 3) + 16 =$$

22.
$$4(2 \times 3) + 24 =$$

23.
$$\frac{5(3-1)}{4\times3^2}$$

24.
$$\frac{8-(2+1)}{5^2}$$

25.
$$\frac{(14-11)+7}{8\times5}$$

Chapter 7 Test

- 1. The variable in 9x + 63 = 198 is
- (A) x.
- (B) 9.
- (C) 63.
- (D) = .
- 2. The coefficient in 8p = 897 is
 - (A) p.
 - (B) 8.
 - (C) 897.
 - (D) =.
- 3. The verbal expression "a number squared then increased by ten" can be written as
 - (A) x + 10.
 - (B) x + 102.
 - (C) $(x^2 + 10)$.
- (D) x.
- 4. Which equation would be used to find the sale price of a television on sale for 25% off \$150?
 - (A) s = 0.25(\$150)
 - (B) $s = \$150 (0.25 \times \$150)$
 - (C) s = 0.25 + \$150
- (D) s 25% = \$150

- 5. Which would not be expressed as x 4?
 - (A) four less than a number
 - (B) four more than a number
 - (C) a number less four
 - (D) a number decreased by four
- 6. Which of the following is an example of the distributive property of multiplication?
 - (A) $7 \times 4 = 4 \times 7$
 - (B) $7 \times (4 \times 3) = (7+4) + (7+3)$
 - (C) $7 \times (4 \times 3) = (7 \times 4) + (7 \times 3)$
 - (D) (7+4)+3=7+(4+3)
- 7. Solve. $(8 \times 3) + 4(12 \div 6)$
 - (A) 32
 - (B) 24
 - (C) 64
 - (D) 192
- 8. "A number halved is sixteen" would be expressed as
 - (A) $\frac{x}{2} = 16$.
 - (B) $x = \frac{16}{2}$.
 - (C) 2x = 16.
 - (D) $\frac{2}{x} = 16$.

9. Mr. Alberghetti opened a bakery. This month his total sales, T, increased by 20% over last month, x. Which expression would show his sales this month?

(A)
$$T = x + 0.2$$

(B)
$$T = 0.2x$$

(C)
$$T = x + 0.2x$$

(D)
$$T + 0.2 = x$$

10. If Shana ran 8 laps more than Kelly, and Devin ran 9 more laps than Kelly; which expression shows Devin's laps in terms of Kelly, k?

(A)
$$k - 9$$

(B)
$$k + 9$$

(C)
$$9 - k$$

- (D) none of the above
- 11. Which of the following is an example of the associative property of addition?

(A)
$$(5+6)+8=5+(6+8)$$

(B)
$$5 \times (6+8) = (5+6) + (5+8)$$

(C)
$$5 + (6 \times 8) = (5 \times 6) + (5 \times 8)$$

(D)
$$5+6=6+5$$

12. "Twice a number divided by five" would be expressed

$$(A) \frac{5x}{2}.$$

(B)
$$(x+2) \div 5$$
.

(C)
$$\frac{2x}{5}$$
.

(D)
$$2x + 5$$
.

Use the information below to answer questions 13 and 14.

Pop's Pizza is having a delivery special. They charge a flat \$0.99 delivery (which is a \$2.00 savings) and have medium one topping pizzas for \$5, m, and large one topping pizzas for \$6, L. Extra toppings are \$0.59 each.

13. Jeff ordered two medium pepperoni pizzas. Which equation best shows his total cost, c?

(A)
$$c = \$0.99 + 2m$$

(B)
$$c = 2(\$0.99) + m$$

(C)
$$0.99 = c + m$$

(D)
$$c + m + m = 0.99$$

14. Maria bought 3 pizzas; 2 medium and 1 large. Which equation best shows her total cost, *c*?

(A)
$$c = \$0.99 + 3m$$

(B)
$$c = \$0.99 + 3L$$

(C)
$$c = \$0.99 + 2m + 3L$$

(D)
$$c = \$0.99 + 2m + L$$

15. Simplify $9 \times (20 \div 4) + 7 =$

16.
$$4(5+1) - 18 + (3 \times 2^2)$$

- (A) 4
- **(B)** 0
- (C) 18
- (D) 36